Fisika Matematika III
Kuliah 9:
Persamaan Differensial Parsial

Hasanuddin

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Jurusan Fisika FMIPA UNTAN

Contoh

$$\nabla^2 u(x,y,z) = \nabla \cdot \nabla u$$

Dalam koordinat kartesian

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

- Persamaan Laplace

$$\nabla^2 u(x,y,z) = 0$$

- Persamaan Poisson

$$\nabla^2 u(x,y,z) = f(x,y,z)$$

- Persamaan diffusi

$$\nabla^2 u(\boldsymbol{r},t) = \frac{1}{\alpha^2} \frac{\partial u(\boldsymbol{r},t)}{\partial t}$$

contoh

- Persamaan Gelombang

$$\nabla^2 u(\boldsymbol{r}, t) = \frac{1}{v^2} \frac{\partial^2 u(\boldsymbol{r}, t)}{\partial t^2}$$

- Persamaan Helmhotz

$$\nabla^2 F + k^2 F = 0$$

- Persamaan Schrodinger

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(x, t) + V\,\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}$$

Properties of field

- Jika tidak ada source (sumber)

$$\nabla . \boldsymbol{v} = 0$$

- Jika tidak ada rotasi

$$curl\ \boldsymbol{v} = 0 \Rightarrow \boldsymbol{v} = \nabla u$$

- Jika tidak ada source dan tidak ada rotasi, maka persamaan berikut berlaku:

$$\nabla . \nabla u = \nabla^2 u = 0$$

Pers. Laplace : Temperatur dalam Plat Logam

Temperatur

$$T(x,y)$$

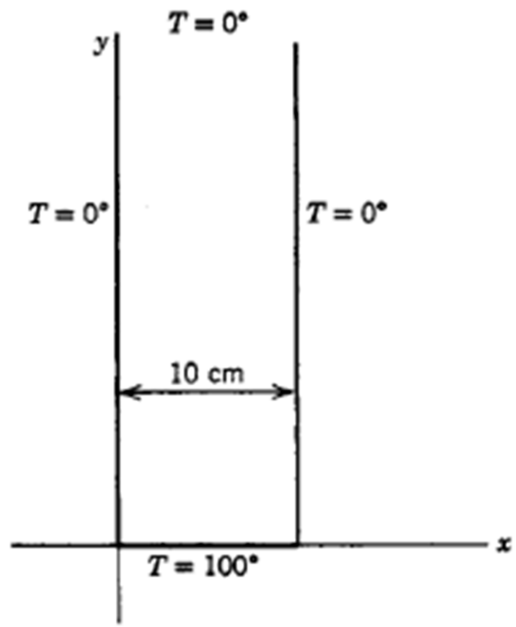
Memenuhi persamaan Laplace:

$$\nabla^2 T = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Trial solution:

$$T = X(x)Y(y)$$



Separasi Variabel

$$\frac{\partial^2 XY}{\partial x^2} + \frac{\partial^2 XY}{\partial y^2} = 0$$

$$Y\frac{d^2 X}{dx^2} + X\frac{d^2 Y}{dy^2} = 0 \quad \times \frac{1}{XY}$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} = 0$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} = -\frac{1}{Y}\frac{d^2 Y}{dy^2} = \text{konstanta} = -k^2$$

Separasi Variabel

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k^2 \quad \Rightarrow \frac{d^2X}{dx^2} = -k^2X$$

$$X'' = -k^2X$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = k^2 \quad \Rightarrow \frac{d^2Y}{dy^2} = k^2Y$$

$$Y'' = k^2Y$$

Solusi

Untuk X:

$$X = \cos kx$$

$$X' = -k \sin kx \quad , \qquad X'' = -k^2 \cos kx$$

Dan

$$X = \sin kx$$

Untuk Y:

$$Y = e^{ky}$$

Dan

$$Y = e^{-ky}$$

Solusi umum:

$$T = XY = (A \cos kx + B \sin kx)\left(Ce^{ky} + De^{-ky}\right)$$

Kondisi Batas

Di batas $y \rightarrow \infty : T = 0$ -> $C = 0$

$$T = (A\cos kx + B\sin kx)e^{-ky}$$

Di batas $x = 0 \rightarrow T = 0.$

$$0 = (A)e^{-ky} \quad \Rightarrow A = 0$$

$$T = B\ e^{-ky}\sin kx$$

$$T = XY = e^{-ky}\sin kx$$

Batas di $x = 10 = L$

$$T(L, y) = 0$$

$$e^{-ky} \sin kL = 0$$

$$\sin kL = 0$$

$$kL = n\pi, \qquad n = 1,2,3, \dots$$

$$k = \frac{n\pi}{L}$$

$$T(x, y) = \sum_{n=1}^{\infty} b_n \, e^{-n\pi y/L} \sin \frac{n\pi}{L} x$$

Batas di $y = 0$

$$T(x,0) = \sum_{n=1}^{\infty} b_n \ \sin\frac{n\pi}{L}x = 100$$

$$b_n = \frac{2}{L}\int_0^L 100\sin\frac{n\pi x}{L}\ dx = \frac{200}{L}\frac{1}{\frac{n\pi}{L}}(-\cos n\pi + \cos 0)$$

$$= \frac{200}{n\pi}(-\cos n\pi + \cos 0)$$

Batas di $y = 0$

$$b_n = \begin{cases} \dfrac{400}{n\pi} & jika\ n\ ganjil \\ 0 & jika\ n\ genap \end{cases}$$

Jadi,

$$T(x,y) = \sum_{n=1}^{\infty} b_n\, e^{-n\pi y/L} \sin\frac{n\pi}{L}x$$

$$T(x,y) = \frac{400}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n\pi y/L}}{n} \sin\frac{n\pi}{L}x$$

Solusi

$$T(x,y) = \frac{400}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{e^{-n\pi y/L}}{n} \sin \frac{n\pi}{L} x$$

$$T(x,y) = \frac{400}{\pi}\left(e^{-\pi y/L} \sin \frac{\pi x}{L} + \frac{1}{3} e^{-\frac{3\pi y}{L}} \sin \frac{3\pi x}{L} + \frac{1}{5} e^{-\frac{5\pi y}{L}} \sin \frac{5\pi x}{L} + \cdots \right)$$

Fourier Series

Sebuah fungsi

$$f(x)$$

Dalam interval $0 < x < L$, mungkin dapat diekspansikan ke dalam deret

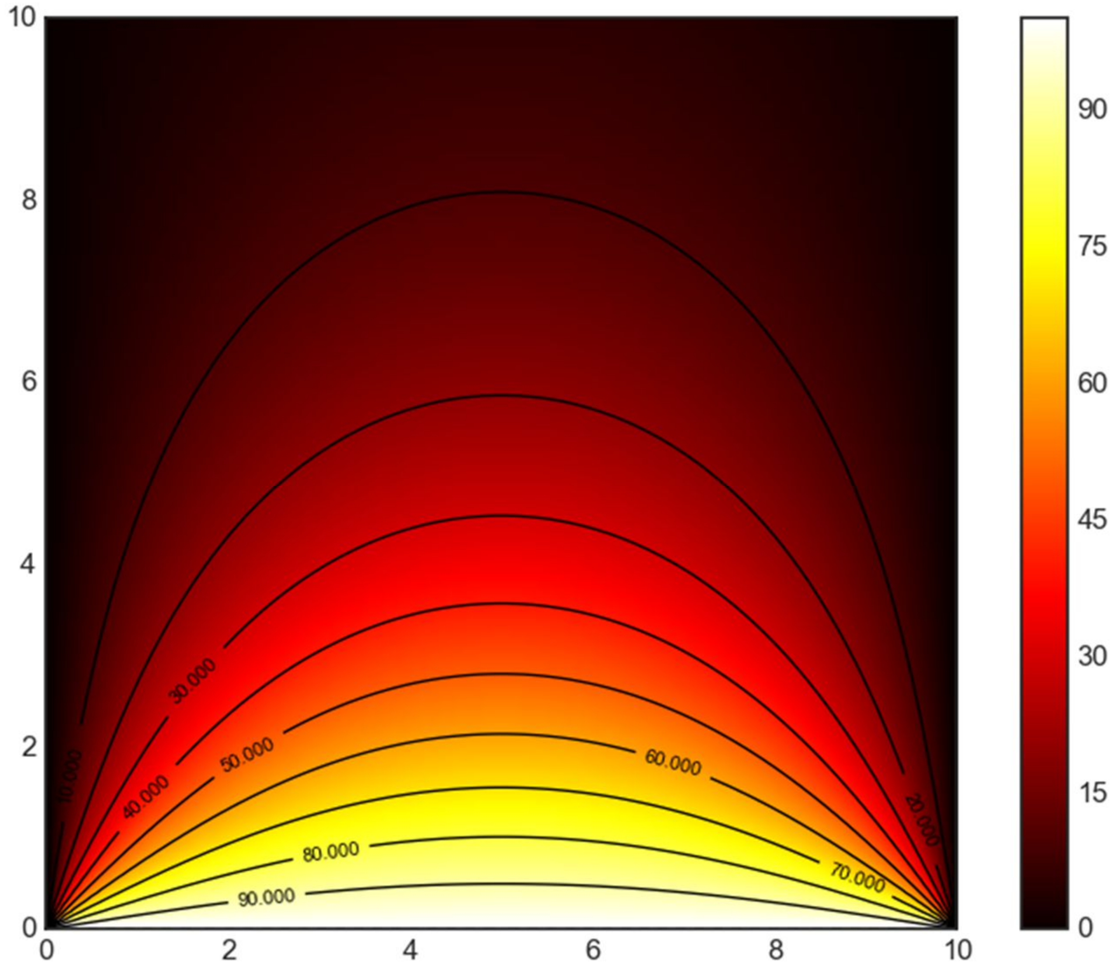
$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$$

Jika $f(x)$ ganjil:

$a_n = 0$ dan

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

Metal Plat



10
8
6
4
2
0
0
2
4
6
8
10
90
75
60
45
30
15
0
30.000
10.000
40.000
50.000
60.000
20.000
80.000
70.000
90.000